CROWDFUNDING WITHOUT INTERMEDIATION?
JUN CHEN†

Abstract. I develop a game-theoretic model to study information asymmetries in the evolving equity crowdfunding market. I assume (1) there are two types of investors: informed (“insiders”) and uninformed (“outsiders”); (2) the insiders invest first; and (3) the outsiders observe the aggregate of insiders’ actions and then decide whether to invest. Under these assumptions, I prove that there does not exist a crowdfunding market equilibrium in which the insiders’ information is aggregated and high quality startups are funded with higher chances. I then use data from Regulation crowdfunding (Title III equity crowdfunding), and provide evidence that is consistent with the model implications. My results suggest that adverse selection is a primary barrier to equity crowdfunding, and new market designs are required to better develop this market.

1. INTRODUCTION

Since crowdfunding first launched in 2008 in the U.S., it has attracted a lot of attention as a financial innovation to alleviate small businesses and startups’ financial constraints.¹ The market has also enjoyed rapid expansion (see Figure 1). To further exploit crowdfunding to increase small businesses and startups’ access to capital, the JOBS Act of 2012 legalized equity crowdfunding (a special type of crowdfunding): investors can now invest online to buy equity shares of startups. The legalization of equity crowdfunding is perceived to be revolutionary for the early-stage financing market because it allows not just accredited investors² to buy unregistered securities online (also referred to as “Title II equity crowdfunding”), but also millions of non-accredited investors to do so (“Title III equity crowdfunding”). This opening of private business investment opportunities to a large number of non-accredited investors is the first such instance since the Security Act of 1933. Distinct from traditional early-stage financing sources, }

¹Division of Humanities and Social Sciences, California Institute of Technology.
²There is a large literature showing that startups and small businesses are financial constrained (e.g., Evans and Jovanovic, 1989; Holtz-Eakin, Joulfaian, and Rosen, 1994), and that access to capital is key to entrepreneurship and innovation (e.g., Schumpeter, 1934; King and Levine, 1993; Brown, Fazzari, and Petersen, 2009).
³According to the SEC, an individual will be considered an accredited investor if he or she: (1) earned income that exceeded $200,000 (or $300,000 together with a spouse) in each of the prior two years, and reasonably expects the same for the current year, or (2) has a net worth over $1 million, either alone or together with a spouse.
such as bank loans and venture capital (VC), the key feature of the equity crowdfunding model is that the capital is directly raised from a large number of investors in relatively small amounts, and no financial intermediation is involved.

Figure 1. THE GROWTH OF CROWDFUNDING

Notes: the following figure (source: Fleming and Sorenson, 2016) reports the worldwide growth of crowdfunding volume by type.

The absence of financial intermediation in the equity crowdfunding market, however, raises questions about how information asymmetries between startups and investors are resolved. In particular, in an environment where the entrepreneurs cannot credibly communicate with the investors, the resolution of information asymmetries requires that information be aggregated from the informed market participants and then conveyed to the uninformed ones. This transmission of information among investors usually relies on price variation in markets (e.g., Grossman and Stiglitz, 1980). However, in the equity crowdfunding market, the offering price is fixed by entrepreneurs ex-ante. Therefore, the price cannot adjust to revealed information. This may thus hinder information aggregation in the equity crowdfunding market. In this paper, I examine the information aggregation issue, and study whether the current equity crowdfunding market rules can resolve one major type of information asymmetry that is common in early-stage financing: adverse selection (the “lemons” problem).

In particular, Title III of the JOBS Act explicitly sets a maximum amount of dollars each investor can annually invest, which should keep the “big players” away from the market and attract mainly small investors. See more at https://www.sec.gov/oiea/investor-alerts-bulletins/ib_crowdfunding-.html
Adverse selection, and information asymmetry more generally, has long been a central issue in economics and finance. For example, Akerlof (1970) first addressed how adverse selection can lead to market failures in automobile and insurance markets. More specific to the environment of early-stage financing, a large literature has focused on the study of how financial intermediaries such as VCs solve information asymmetries. In the setting of equity crowdfunding, adverse selection is a key concern for academics (Catalini, Fazio, and Murray, 2016) and the regulatory authorities. For example, the SEC made this issue clear in the final implementation rule of Regulation Crowdfunding (Title III):

“The statute and the final rules related to entrepreneur disclosures are intended to reduce the information asymmetries that currently exist between small businesses and investors[...]. These considerations may give rise to adverse selection and moral hazard in offerings in reliance on Section 4(a)(6).”

To study the adverse selection issue in equity crowdfunding, I develop a simple game-theoretic model. In the model, I assume that there are two types of investors: informed (“insiders”) and uninformed (“outsiders”). The insiders would include entrepreneurs’ family and friends (“F&F”), social network friends, and maybe some local angel investors. The informational advantage of F&F and social network friends has been well documented in early-stage financing. The outsiders in my model are investors who are outside the entrepreneurs’ personal and social circles, and only learn about startups from the crowdfunding platforms. I exogenously fix the order of how insiders and outsiders take investment actions. In particular, I assume that the insiders take their investment actions first, and the outsiders observe the aggregate of the insiders’ actions and then decide whether to invest or not. Empirical evidence supports this assumption. For example, Agrawal, Catalini, and Goldfarb (2011) show that F&F accounts for a big proportion of the early investors. In fact, it has become an important practice for entrepreneurs on the Kickstarter platform to leverage their own network:

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4For example, Amit, Brander, and Zott (1998) argue that the very existence of VCs relies on their ability to reduce the costs of information asymmetry. Chan (1983) develops a theory of financial intermediation that highlights the contribution of intermediaries as informed agents in a market with imperfect information. To overcome the information asymmetry problem, the intermediaries also usually develop some special mechanisms. For example, the VC firms send their representatives to sit on the boards of the private firms they have invested in (Lerner, 1995), and also usually structure their investments in stages (Gompers, 1995).


"The first thing they tell you at Kickstarter is to leverage your own network. But people are like, I came to Kickstarter so they would give me free money!"

-Page 82, Steinberg (2012), The Kickstarter Handbook

Then using my model, I investigate whether there is a crowdfunding market equilibrium in which an entrepreneur’s expected utility is maximized, the insiders’ information is effectively aggregated, and the outsiders’ participation constraint is satisfied. At the crowdfunding market equilibrium, the market would be able to differentiate high quality startups from low quality ones, and fund high quality ones with a significantly higher probability, thus alleviating the adverse selection problem. However, my main result in this paper shows that there does not exist such a crowdfunding market equilibrium. The primary driver of my result is the incompatibility of aggregating insiders’ information and satisfying outsiders’ participation constraint when the share price is fixed for all investors.

The non-existence of a crowdfunding market equilibrium has several implications for the equity crowdfunding market. First, a decentralized equity crowdfunding market may fail to overcome the “lemons” problem that plagues early-stage financing. More specific to Title III equity crowdfunding in which investment vehicles are prohibited, the adverse selection problem may be a key factor limiting the growth of that market. Second, to promote the development of the equity crowdfunding market, both practitioners and policy makers may need to focus on the adverse selection problem, and develop new market mechanisms to address it. For example, as discussed in Section 5 below, mechanisms allowing for varying prices during the offering process solve the outsiders’ participation constraint problem. Additionally, it may be helpful to encourage the crowdfunding portals to take more responsibility in screening startups listed in their platforms.

This paper provides the first rigorous study of the adverse selection issue in the equity crowdfunding market. In particular, the equity crowdfunding market involves transactions of equity stakes, and thus is more formal than other types of crowdfunding models such as reward-based crowdfunding. The relatively formal nature of the equity crowdfunding market allows me to model its market participants as rational agents, and examine the participants’ interaction from an information aggregation perspective. With this framework, I find that one of the major early-stage financing market frictions, adverse selection, persists in the equity crowdfunding market, and may not be solved by market mechanisms under current market designs. My paper provides insights on understanding the equity crowdfunding market in a rigorous economic
framework, and also sheds light on policy implications for how to promote the development of the equity crowdfunding market.

My paper relates to several literatures. First, it directly relates to a small literature studying information asymmetries in the environment of equity crowdfunding. For example, Agrawal, Catalini, and Goldfarb (2016) use data from AngelList and argue that syndicates of investors are effective tools in reducing information asymmetries in equity crowdfunding. My paper complements theirs by providing a rigorous analysis for the setting in which no financial intermediation, such as syndicates, are involved. In this regard, my paper speaks more directly to Title III equity crowdfunding where the law prohibits financial intermediation such as syndicates.

Second, my paper is broadly related to the literature studying the role of price in conveying information from the informed market participants to the uninformed. For example, Grossman and Stiglitz (1980) argue that price cannot perfectly reflect the information that is available in the market, because the price must be such that the informed participants receive compensation for their cost of acquiring information. Similar to theirs, my paper shows that when the market price is fixed for all participants regardless of their information, no market equilibrium exists that can effectively convey the information from the informed to the uninformed.

Third, my paper is also related to papers investigating the “wisdom of the crowd” in crowdfunding markets. For example, Hakenes and Schlegel (2014) argue that the “all or nothing” mechanism can be used by (homogenous) household investors to aggregate information so that at equilibrium they acquire information and the aggregation of their information enables more good projects to receive funding. The focus of their paper is on how firms and investors interact to facilitate crowdfunding. Different from their paper, I focus on the study of information asymmetry/adverse selection and the interaction between heterogeneous investors. Another related paper is Brown and Davies (2015), who build a one-period model with naive and sophisticated investors. In their model, identical naive investors have weak information and act on the information, and sophisticated investors receive private information and behave strategically. The main conclusion of their paper is that naive investors rather than sophisticated ones communicate the “wisdom of the crowd” and improve financing efficiency.

Fourth, my paper also connects to the literature studying quality signaling of online markets with asymmetric information. For example, Bernstein, Korteweg, and Laws (2017) show that average investors respond strongly to the founding team. Mollick (2014) argues that the entrepreneurs’ social capital and preparedness are associated with an increased chance of project
success, suggesting that quality signals play a role in project outcomes. These papers are related to mine in the sense that I assume that even if the entrepreneurs have information concerning the quality of their startups, they will not have credible channels to signal this information to the investors. Therefore, the remaining information asymmetries are still high. Unlike other online markets, a crowdfunding campaign is usually a one-shot game, which also invalidates many signaling mechanisms, such as reputation signaling, that are effective in other markets (Agrawal, Catalini, and Goldfarb, 2014).

Finally, my paper also speaks to the finance literature on information cascades and investor herding. A major focus of that literature is to explain the occurrence of herding and its consequences (see e.g., Bikhchandani, Hirshleifer, and Welch, 1992; Welch, 1992). In the crowdfunding market, some studies also show the existence of investors’ herding behavior (see e.g., Agrawal, Catalini, and Goldfarb, 2011; Zhang and Liu, 2012). In particular, Agrawal, Catalini, and Goldfarb (2011) show that investors are much more likely to invest in startups that have reached a higher percentage of their funding goal. My paper differs from these papers in several aspects. First, unlike the case in which investors make decisions sequentially in the information cascade literature, in my model the investors with private information do not move sequentially, preventing information cascades from happening. Second, in my model it is part of the uninformed investors’ investing strategy to invest in those startups that have already attracted more investors. For the uninformed investors, the number of existing investors is a public signal from which they infer the quality of startups. In this sense, the uninformed investors’ behavior cannot be viewed as herding in the classical case.

The paper is organized as follows. In Section 2, I introduce the formal model. In Section 3, I analyze the market equilibrium, and prove the main result. In Section 4, I provide empirical evidence for my model’s implications. In Section 5, I suggest an improving market mechanism. I conclude in Section 6.

2. The model

Consider an equity crowdfunding campaign in which an entrepreneur is raising capital for her startup. The startup is of high or low quality, denoted by $H$ and $L$, respectively. The common prior on the startup’s quality is $P(H) = P(L) = 1/2$, known to the entrepreneur and all investors. The entrepreneur wants to sell a fixed number of shares of her startup. In practice, when issuing equity, the entrepreneur can choose two things to reflect how much she evaluates her startup:
share price and number of shares to sell. For simplicity, in my model, I fix the number of shares (or the fraction of the equity placed on the market), and only allow the entrepreneur to choose share price (denoted by \( p \)).

I assume a so-called “all or nothing” rule in my model for determining the final outcome of an equity crowdfunding campaign: the entrepreneur receives capital from the crowdfunding campaign if and only if there are enough investors to buy all the shares offered for sale. This rule is consistent with Title III equity crowdfunding implementation rule.\(^7\) So the entrepreneur either sells all or none of the shares she offers. I refer to the former case as “a successful crowdfunding campaign” (denoted by \( S \)). Only upon a successful crowdfunding campaign are the entrepreneur and investors’ payoffs in the crowdfunding market realized.

I assume that the entrepreneur does not know the quality of her startup. This assumption is technically equivalent to the scenarios in which either the entrepreneur does not have credible mechanisms to signal her startup’s quality even if she knows it, or the entrepreneur cannot accurately estimate the market prospects of her startup. The entrepreneur is an expected utility maximizer with Bernoulli utility \( u(x) = x^\gamma \), \( \gamma \in (0, 1] \). Because the entrepreneur has a fixed number of shares to sell, as long as she maximizes her expected utility from selling just one share, she maximizes her utility for the whole sale. So the entrepreneur solves the following optimization problem:

\[
\max_p P(S) \cdot p^\gamma = \max_p \frac{P(S|H) + P(S|L)}{2} \cdot p^\gamma
\]  

\(^7\)The Title III final rule states that: “...including a statement that if the sum of the investment commitments does not equal or exceed the target offering amount at the offering deadline, no securities will be sold in the offering, investment commitments will be cancelled and committed funds will be returned.”
All the investors are rational, and the insiders behave strategically. For simplicity, I assume that each investor can buy at most one share. Then each investor’s strategy boils down to a pure strategy: either invest or not, or a mixed strategy of investing and not investing. Individual investors’ investing strategies depend on their information. The insiders’ investing strategy depends on their private signals and the share price set by the entrepreneur. The outsiders’ investing strategy depends on the number of insiders who invest and also the share price. The return of one share is 1 when the startup quality is “High”, and 0 otherwise. All investors are risk neutral with Bernoulli utility \( u(x) = x \), and they are expected utility maximizers. The investors are assumed to be risk neutral, because in the crowdfunding setting, each investor is required to just invest a small amount of her money into the market, and to be prepared for the possible loss of all that money.\(^8\) Let \( Y \) denote the action that an investor invests, and let \( \mathbb{E}^I(Y|F,S) \) denote the expected utility (payoff) of an insider who receives an \( F \) signal (also termed as an “\( F \)-signal insider”) (\( F = G, B \)). So for \( F = G \) or \( B \),

\[
\mathbb{E}^I(Y|F,S) = \mathbb{P}(H|F,S) \times 1 + \mathbb{P}(L|F,S) \times 0 - p = \mathbb{P}(H|F,S) - p. \tag{2}
\]

Let \( n_I \) denote the number of insiders who invest. The outsiders’ expected utility (payoff) is:

\[
\mathbb{E}^O(Y|n_I,S) = \mathbb{P}(H|n_I,S) \times 1 + \mathbb{P}(L|n_I,S) \times 0 - p = \mathbb{P}(H|n_I,S) - p. \tag{3}
\]

\(^8\)Since there are infinitely many identical outsiders, they are determined randomly to invest in a startup if too many of them want to invest.

\(^9\)The SEC sets a clear rule on the limit an investor can invest in the equity crowdfunding market each year. The limit depends on the investors’ income, but the general principle is that an investor can absorb the risk of losing all the money she invests.
3. The Crowdfunding market equilibrium

3.1. Main result. In this section, I analyze the formal model and prove my main result. I focus on symmetric strategies equilibria. By playing symmetric strategies, the same type of investors (insiders and outsiders) use identical strategies if they have the same information set. Then I define a market equilibrium that is desired for a well-functioning crowdfunding market:

Definition 1. A crowdfunding market equilibrium is an equilibrium where

1. the entrepreneur chooses share price \( p \) to maximize her expected utility,
2. the insiders with bad (B) signals do not invest, and
3. the outsiders’ participation constraint is satisfied.

I require that the B-signal insiders do not invest, because only when not all insiders invest can the insiders’ information be effectively aggregated, and high quality startups can achieve significantly higher success rates of raising capital than low quality ones. Also, only in this case can the adverse selection problem in the crowdfunding market be overcome. As Lemma 3 below shows, as the insiders’ likelihood of investing increases (e.g., more and more B-signal insiders start to invest), high quality startups’ chances of crowdfunding success fall relative to low quality ones’.

It has long been recognized that adverse selection could lead to markets failure (Akerlof, 1970), which also makes overcoming this issue essential for the crowdfunding market. However, my main result shows that in the absence of a financial intermediary, the decentralized crowdfunding market may not be able to do so.

Theorem 1. There does not exist a crowdfunding market equilibrium.

It has been widely expected that crowdfunding democratizes the investing opportunities to all investors, and reduces geography-related frictions in early-stage investing (e.g., Kim and Hann, 2013). However, my result in Theorem 1 shows that the crowdfunding market could fall short of people’s expectations. In the rest of this section, I analyze the formal model and present the main steps to prove Theorem 1.

3.2. Outsiders’ equilibrium strategies. I first characterize outsiders’ equilibrium strategies. I say that the outsiders play cutoff strategies if there exists a threshold number \( n_0 + 1 \) such
that outsiders invest in a startup if and only if they observe that at least \( n_0 + 1 \) insiders have invested. Then I prove the following result.

**Proposition 1.** At equilibrium, outsiders play cutoff strategies.

The proof of Proposition 1 is in Appendix A. The key to proving Proposition 1 is to first narrow down insiders’ strategy space, and then show that, conditioning on the insiders’ strategy space, the outsiders play cutoff strategies. To narrow down insiders’ strategy space, I show that insiders always receive a higher expected payoff from investing if the private signal is more positive.

**Lemma 1.** \( \mathbb{E}^I(Y|G, S) > \mathbb{E}^I(Y|B, S) \).

The proof of Lemma 1 follows from the Bayes’ rule and the independence of private information among insiders. Lemma 1 separates the \( G \)-signal insiders’ investing behavior from the \( B \)-signal ones, and it restricts the insiders’ strategy space to five cases.

1. **Case I:** no insider invests.
2. **Case II:** \( G \)-signal insiders use mixed strategy to invest, \( B \)-signal insiders do not invest.
3. **Case III:** \( G \)-signal insiders invest, \( B \)-signal insiders do not invest.
4. **Case IV:** \( G \)-signal insiders invest, \( B \)-signal insiders use mixed strategy to invest.
5. **Case V:** all insiders invest.

Once the insiders’ strategy space is specified, it suffices to prove that the outsiders play a cutoff strategy conditioning on each possibility of insiders’ strategy. See details in the proof of Proposition 1.

3.3. **Insiders’ equilibrium strategies.** Next I analyze insiders’ equilibrium investing strategy. I show that conditioning on the outsiders’ cutoff strategies \( n_0 + 1 \) and a fixed share price \( p \), the insiders have unique symmetric equilibrium strategies. Furthermore, I compute insiders’ equilibrium strategies in terms of share price \( p \).

**Proposition 2.** Conditioning on the outsiders’ cutoff strategies \( n_0 + 1 \) and a fixed share price \( p \), there exist unique insiders’ symmetric equilibrium strategies. Moreover, for fixed parameters \( \alpha \), \( n \), and \( n_0 \), the equilibrium strategies can be characterized according to the share price \( p \). Denote

\[
\varepsilon(n, n_0, \alpha) = \frac{\sum_{k=n_0}^{n} \binom{n}{k}(1-\alpha)^k \alpha^{n-k}}{\sum_{k=n_0}^{n} \binom{n}{k}\alpha^k(1-\alpha)^{n-k}}.
\]

The insiders’ equilibrium strategies have the following forms:
I. If \( p \geq \frac{1}{1+\left(\frac{1}{\alpha}\right)^{n_0+1}} \), then at the equilibrium, no insider invests (referred to as Equilibrium I).

II. If \( \frac{1}{1+\frac{1}{\alpha}e(n,n_0,\alpha)} < p < \frac{1}{1+\left(\frac{1}{\alpha}\right)^{n_0+1}} \), then at the equilibrium, the insiders with G signals use a mixed strategy to invest and those with B signals do not invest (referred to as Equilibrium II).

III. If \( \frac{1}{1+\frac{1}{\alpha}e(n,n_0,\alpha)} \leq p \leq \frac{1}{1+\left(\frac{1}{\alpha}\right)^{n_0+1}} \), then at the equilibrium, all insiders with G signals invest and those with B signals do not (referred to as Equilibrium III).

IV. If \( 1-\alpha < p < \frac{1}{1+\frac{1}{\alpha}e(n,n_0,\alpha)} \), then at the equilibrium, the insiders with G signals invest and those with B signals use a mixed strategy to invest (referred to as Equilibrium IV).

V. If \( p \leq 1-\alpha \), then at the equilibrium, all the insiders invest regardless of their private signals (referred to as Equilibrium V).

The proof of Proposition 2 is in Appendix A. Because Cases I and V can be considered special cases of Cases II and IV, respectively, and Case III is an intermediate case, I just need to prove Proposition 2 for Cases II and IV. At equilibrium, when the insiders play mixed strategies, they are indifferent between investing and not investing. So at Case II, a G-signal insider receives zero expected payoff from investing, i.e.

\[
\mathbb{E}^I(Y|G,S) = \mathbb{P}(H|G,S) - p = 0.
\]  

(4)

Similarly, at Case IV, a B-signal insider receives zero expected payoff from investing, i.e.

\[
\mathbb{E}^I(Y|B,S) = \mathbb{P}(H|B,S) - p = 0.
\]  

(5)

Then conditioning on the outsiders’ cutoff strategies \( n_0 + 1 \) and a fixed share price \( p \), to prove the uniqueness of insiders’ equilibrium strategies, I just need to prove that there is a unique solution of insiders’ strategies to the equilibrium conditions (4) and (5). Equivalently, I just need to prove that one insider’s gross payoff (\( \mathbb{P}(H|G,S) \) and \( \mathbb{P}(H|B,S) \)) is monotone in her mixed probability of investing when assuming all other investors (both insiders and outsiders) play equilibrium strategies. For Case II, let \( r_G \) denote the mixed probability that a G-signal insider invests. Similarly, for Case IV, let \( r_B \) denote the mixed probability that a B-signal insider invests. Then I prove the following result.

**Lemma 2.** (1) At Equilibrium II, one G-signal insider’s gross payoff \( \mathbb{P}(H|G,S) \) is strictly decreasing in \( r_G \in (0,1] \).
(2) *At Equilibrium IV*, one B-signal insider’s gross payoff $\mathbb{P}(H|B,S)$ is strictly decreasing in $r_B \in [0, 1]$.

The proof of Lemma 2 is in Appendix A. Although the proof of Lemma 2 is a bit involved, the intuition is simple. For example, at Equilibrium II, when the insiders with good signals increase their investing probability, the probability that at least $n_0 + 1$ insiders invest in a low quality startup increases relatively more than that probability in a high quality startup. Therefore, a low quality startup has relatively higher probability of being successfully funded when the $G$-signal insiders invest more aggressively, thus reducing $G$-signal insiders’ gross expected payoff at equilibrium.

### 3.4. Effective screening

As argued before, whether the crowdfunding market can avoid market failure relies on its ability to solve the adverse selection problem. More specifically, it depends on whether the informed investors’ information can be aggregated effectively. In this section, I show that the effectiveness of insiders’ information aggregation decreases in insiders’ likelihood of investing.

**Definition 2.** Define the effectiveness of insiders’ information aggregation as the ratio of the probability that a high quality startup gets funded over the probability that a low quality one does so:

$$IA = \frac{\mathbb{P}(S|H)}{\mathbb{P}(S|L)}.$$

**Lemma 3.** The effectiveness of insiders’ information aggregation $IA$ is decreasing in insiders’ likelihood of investing. In particular, $IA$ is decreasing in $r_G$ at Equilibrium II and decreasing in $r_B$ at Equilibrium IV.

The proof of Lemma 3 is in the Appendix. Lemma 3 demonstrates that as insiders become more likely to invest, the crowdfunding market’s relative ability to fund high quality startups weakens, and the crowdfunding market becomes less effective in screening startups. This also motivates the second condition in the definition of crowdfunding market equilibrium that $B$-signal insiders do not invest. Under the condition that $B$-signal insiders do not invest, I just need to focus on the first three cases of insiders’ equilibrium strategies (Equilibrium I, II, III) when investigating the existence of a crowdfunding market equilibrium and solving the entrepreneur’s problem.
3.5. The entrepreneur’s equilibrium strategy. From the standpoint of the entrepreneur, when conditioning on the outsiders’ equilibrium strategies, the event \( S \) (a successful crowdfunding campaign) is equivalent to that at least \( n_0 + 1 \) insiders invest. So the entrepreneur’s problem becomes:

\[
\max_p \frac{\mathbb{P}(n_I \geq n_0 + 1|H) + \mathbb{P}(n_I \geq n_0 + 1|L)}{2} \cdot p^\gamma
\]

Let \( n_i = n_I - 1 \). Fix one insider’s investing decision. Then from this insider’s point of view, the event \( n_I \geq n_0 + 1 \) is also equivalent to the event that at least \( n_0 \) other insiders invest, i.e. \( n_i \geq n_0 \). Because the share price \( p \) is a function of \( \mathbb{P}(n_i \geq n_0|H) \) and \( \mathbb{P}(n_i \geq n_0|L) \) at equilibrium (see details in the proof of Proposition 2), it is convenient to solve an equivalent problem of the entrepreneur:

\[
\max_p \frac{\mathbb{P}(n_i \geq n_0|H) + \mathbb{P}(n_i \geq n_0|L)}{2} \cdot p^\gamma
\]

(6)

Because the expressions of \( \mathbb{P}(n_i \geq n_0|H) \), \( \mathbb{P}(n_i \geq n_0|L) \) and \( p \) vary across different insiders’ equilibrium strategy cases, to solve the entrepreneur’s problem, it is useful to take a two-step procedure: (1) find an optimal price at each insiders’ equilibrium strategy case, and (2) look for the global maximum across all insiders’ equilibrium strategy cases. As discussed before, it suffices to focus on the first three cases of insiders’ equilibrium strategies (Equilibrium I, II, III). Therefore, (6) is equivalent to the following problem:

\[
\max_{\text{Equilibrium I, II, III}} \max_p \frac{\mathbb{P}(n_i \geq n_0|H) + \mathbb{P}(n_i \geq n_0|L)}{2} \cdot p^\gamma
\]

I can further simplify the above problem. In the first case of the insiders’ equilibrium strategies, no insider invests, thus the entrepreneur’s expected utility is zero. In the third case, the insiders who invest are those with \( G \) signals. Because the number of \( G \) signals does not depend on the price \( p \), \( \mathbb{P}(n_i \geq n_0|H) \) and \( \mathbb{P}(n_i \geq n_0|L) \) do not depend on \( p \). Thus, in the third case, the entrepreneur’s expected utility is maximized at the highest price possible \( p^* = \frac{1}{1 + \frac{1}{\alpha \cdot e(n, n_0, \alpha)}} \). At \( p^* \), the insiders’ equilibrium strategies can also be considered a special case of Equilibrium II at \( r_G = 1 \). Therefore, I just need to solve the entrepreneur’s problem at the insiders’ equilibrium strategy at Equilibrium II. Then I prove the following result.
**Proposition 3.** For any $\gamma \in (0, 1]$, at insiders’ equilibrium strategies II (Equilibrium II), the entrepreneur’s expected utility is maximized at the share price $p^* = \frac{1}{1 + \frac{\gamma^{n_{0}}}{\alpha^{n,n_{0},\alpha}}} \cdot e^{(n,n_{0},\alpha)}$, which corresponds to the insiders’ equilibrium strategy $r_G = 1$.

The proof of Proposition 3 is in Appendix A. Proposition 3 implies that conditioning on insiders’ equilibrium strategy at Equilibrium II and outsiders’ equilibrium strategies at $n_0 + 1$, the entrepreneur’s expected utility is maximized at $p^* = \frac{1}{1 + \frac{\gamma^{n_{0}}}{\alpha^{n,n_{0},\alpha}}} \cdot e^{(n,n_{0},\alpha)}$. At $p^*$, $G$-signal insiders invest for sure and $B$-signal insiders do not. Then taking all three cases (Equilibrium I, II, III) together, we have that when $B$-signal insiders do not invest and all investors play equilibrium strategies, the entrepreneur will set the share price at $p^*$ to maximize her expected utility.

**3.6. Violation of outsiders’ participation constraint.** At a market equilibrium, the outsiders’ participation constraint has to be satisfied. In other words, the outsiders’ expected payoff has to be non-negative at equilibrium. Because the outsiders play a cutoff strategy at equilibrium, it suffices to guarantee that the outsiders receive a non-negative expected payoff at the threshold $n_0 + 1$. However, I show that if the entrepreneur sets the share price at $p^*$, the outsiders have a strictly negative expected payoff at the threshold of any equilibrium strategy.

**Proposition 4.** For fixed parameters $n, n_0$, and $\alpha$, if the entrepreneur sets the share price at $p^* = \frac{1}{1 + \frac{\gamma^{n_{0}}}{\alpha^{n,n_{0},\alpha}}} \cdot e^{(n,n_{0},\alpha)}$, then the outsiders’ expected payoff at the threshold $n_0 + 1$ of any equilibrium strategy is strictly negative.

The proof of Proposition 4 is in Appendix A. Proposition 4 implies that for a given funding threshold $n_0 + 1$ of the outsiders, when the entrepreneur chooses the optimal price and insiders play equilibrium strategies, it is not profitable for the outsiders to actually participate right at the threshold. Proposition 4 leads to the proof of the main result of Theorem 1.

**4. Empirical Evidence**

In this section, I use data from Regulation Crowdfunding (Regulation CF, or Title III equity crowdfunding), and provide evidence that is consistent with my model’s implications. The data period is between the start of Title III equity crowdfunding in May 2016 until September 2017.
My final sample includes 461 firms’ crowdfunding filings\(^{10}\) and 133 successful crowdfunding offerings. See Table 1 for summary statistics of the data.

Analyzing the data, I find several pieces of evidence that are consistent with my model’s implications. First, I find that the total number of Title III crowdfunding offerings is small. In contrast to Regulation CF, Regulation D, the current dominant private offering regulation, attracted 27,725 firms to use it during the same period (05/16/2016-9/3/2017). So in terms of number of firms using each regulation, the Regulation D market is currently 60 times larger than the Regulation CF market. In this regard, after being legal more than one year, the Regulation CF market has grown much more slowly than expected. We can also look at the Regulation CF market in terms of sectors. The sector with the greatest number of successful crowdfunding offerings is the food and beverage industry, especially small breweries and restaurants.\(^{11}\) Upon launching crowdfunding offerings, the firms from the food and beverage industries also have higher success rates of completing their offering than the technology sectors.\(^{12}\) Therefore, the overall development of the Regulation CF market suggests that the market size is small, and the market has favored the industries that potentially have lower information asymmetries, such as restaurants. This pattern is consistent with my model’s implication that the Regulation CF market is not able to solve the “lemons” problem, and thus performs worse in industries with higher information asymmetries.

Second, in terms of financials, the Regulation CF market does not appear to fund high quality firms with higher probability. To examine whether the current Regulation CF market is able to fund high quality firms, I investigate the link between firms’ financials and their rate of succeeding in their crowdfunding offerings. In particular, all else equal, firms with better financials e.g. higher profitability, should have a higher rate of completing their crowdfunding offerings. To test this, I use firms’ net income in the most recent fiscal year to measure firms’ profitability, and explore the following regression framework:

\[
Y_i = \delta_0 + \delta_1 I_i + \delta_2 X_i + \alpha_p + \alpha_L + \epsilon_i
\]

\(^{10}\)The data are collected from the SEC Edgar website: https://www.sec.gov/edgar/searchedgar/companysearch.html. A few firms have multiple crowdfunding campaigns, in which case I only include the first one.

\(^{11}\)Using the first six months of data, 28.13% of all funded campaigns are from food and beverage companies, see https://blog.vcexperts.com/2016/12/20/regulation-crowdfunding-a-six-month-update/.

\(^{12}\)In the first six months, the food and beverage sector accounts for 17.5% of all launched campaigns, and 28.13% of all funded campaigns, in contrast to the technology sector which accounts for 25% of all launched campaigns, and 12.5% of all funded campaigns, see https://blog.vcexperts.com/2016/12/20/regulation-crowdfunding-a-six-month-update/.
Table 1. SUMMARY STATISTICS

Notes: The table reports the summary statistics of Regulation Crowdfunding filings up to September 3, 2017. The sample includes each firm’s first crowdfunding campaign. The variables include: “Compensation %”, the percentage of offering size charged by funding portals upon a successful offering; “Total asset”, the total asset of a firm in the most recent fiscal year; “Cash”, the total cash and cash equivalents in the most recent fiscal year; “Corporation”, a dummy variable indicating whether a firm is incorporated; “Campaign Success”, a dummy variable whether a crowdfunding offering is successful. The units of some variables are in million dollars (M).

<table>
<thead>
<tr>
<th>Variable</th>
<th>mean</th>
<th>sd</th>
<th>min</th>
<th>p25</th>
<th>p50</th>
<th>p75</th>
<th>max</th>
<th>count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age at financing</td>
<td>2.48</td>
<td>3.55</td>
<td>0.01</td>
<td>0.36</td>
<td>1.38</td>
<td>3.28</td>
<td>45.07</td>
<td>459</td>
</tr>
<tr>
<td>Compensation %</td>
<td>5.36</td>
<td>1.86</td>
<td>0.00</td>
<td>4.00</td>
<td>5.00</td>
<td>6.00</td>
<td>12.00</td>
<td>459</td>
</tr>
<tr>
<td>Offering amount (M)</td>
<td>0.10</td>
<td>0.13</td>
<td>0.01</td>
<td>0.03</td>
<td>0.05</td>
<td>0.10</td>
<td>1.00</td>
<td>459</td>
</tr>
<tr>
<td>Number of employees</td>
<td>5.24</td>
<td>8.85</td>
<td>0.00</td>
<td>1.00</td>
<td>3.00</td>
<td>6.00</td>
<td>100.00</td>
<td>459</td>
</tr>
<tr>
<td>Total asset (M)</td>
<td>0.38</td>
<td>1.52</td>
<td>0.00</td>
<td>0.00</td>
<td>0.04</td>
<td>0.23</td>
<td>21.28</td>
<td>459</td>
</tr>
<tr>
<td>Cash (M)</td>
<td>0.07</td>
<td>0.19</td>
<td>-0.06</td>
<td>0.00</td>
<td>0.00</td>
<td>0.05</td>
<td>2.32</td>
<td>459</td>
</tr>
<tr>
<td>Revenue (M)</td>
<td>0.30</td>
<td>0.93</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.08</td>
<td>7.45</td>
<td>459</td>
</tr>
<tr>
<td>Net income (M)</td>
<td>-0.22</td>
<td>0.61</td>
<td>-5.78</td>
<td>-0.15</td>
<td>-0.01</td>
<td>0.00</td>
<td>0.39</td>
<td>459</td>
</tr>
<tr>
<td>Common stock</td>
<td>0.34</td>
<td>0.47</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
<td>459</td>
</tr>
<tr>
<td>Debt</td>
<td>0.23</td>
<td>0.42</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
<td>459</td>
</tr>
<tr>
<td>Corporation</td>
<td>0.71</td>
<td>0.46</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
<td>459</td>
</tr>
<tr>
<td>Incorporated in CA</td>
<td>0.14</td>
<td>0.34</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
<td>459</td>
</tr>
<tr>
<td>Incorporated in DE</td>
<td>0.46</td>
<td>0.50</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
<td>459</td>
</tr>
<tr>
<td>Campaign Success</td>
<td>0.29</td>
<td>0.45</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
<td>459</td>
</tr>
</tbody>
</table>

Here, \( Y_i \) is a dummy variable indicating whether firm \( i \) has successfully completed its crowdfunding offering. The variable of interest \( I_i \) is the net income of firm \( i \) in the most recent fiscal year. \( X_i \) includes firm level controls such as age at crowdfunding, number of employees, and so on. These controls take care of firms’ heterogeneity effects in the corresponding dimensions that relate to firms’ success rate of crowdfunding offerings. Importantly, equation (7) also allows me to control crowdfunding portal fixed effects (\( \alpha_p \)). Different crowdfunding portals appeal to different sets of investors, and also have different volumes of active investors, so launching crowdfunding campaigns in different portals could significantly impact firms’ chances of success. Therefore, controlling portal fixed effects is important in the regression. I also include several other fixed effects, such as firms’ residence and incorporation location (\( \alpha_L \)) in the regression.

Table 2 reports regression results of (7). Although Column (1) shows a significant correlation between firms’ net income and their success rate of crowdfunding, the significance disappears once I control for revenue (Column (2)). Moreover, if we focus on successful crowdfunding that raised more than 0.5 million dollars in Columns (3) and (4), the positive correlation completely disappears. These results suggest that firms’ profitability does not predict firms’ success in crowdfunding. In other words, the crowdfunding market does not seem to be financing high
Table 2. Firms’ Financials and Success of Crowdfunding

Notes: This table reports results from OLS regressions of (7). The dependent variables are “Success”: a dummy variable indicating whether a firm has successfully completed its crowdfunding offering, and “Big Success”, a dummy variable indicating whether a firm has successfully completed its crowdfunding offering with capital raised greater than $500000. The main explanatory variable is “Net income”, the net income of a firm in the most recent fiscal year. Several controls are “Revenue”, the revenue of a firm in the most recent fiscal year, “Total asset”, total assets of a firm in the most recent fiscal year, “Cash”, cash and cash equivalents in the most recent fiscal year. For some of the explanatory variables, the units are in million dollars (M). The sample includes firms’ first crowdfunding campaign between 05/16/2016-9/3/2017. Robust standard errors are reported in parentheses. Significance: * p < 0.10, ** p < 0.05, *** p < 0.01.

<table>
<thead>
<tr>
<th></th>
<th>Success</th>
<th>Big Success</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Net income (M)</td>
<td>0.064*</td>
<td>0.063</td>
</tr>
<tr>
<td>(0.038)</td>
<td>(0.039)</td>
<td>(0.033)</td>
</tr>
<tr>
<td>Revenue (M)</td>
<td>0.005</td>
<td>0.079***</td>
</tr>
<tr>
<td>(0.036)</td>
<td>(0.025)</td>
<td></td>
</tr>
<tr>
<td>Total asset (M)</td>
<td>-0.002</td>
<td>-0.003</td>
</tr>
<tr>
<td>(0.012)</td>
<td>(0.012)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>Cash (M)</td>
<td>0.326***</td>
<td>0.321***</td>
</tr>
<tr>
<td>(0.108)</td>
<td>(0.118)</td>
<td>(0.088)</td>
</tr>
<tr>
<td>Number of employees</td>
<td>0.006**</td>
<td>0.006*</td>
</tr>
<tr>
<td>(0.002)</td>
<td>(0.003)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Age at financing</td>
<td>0.004</td>
<td>0.004</td>
</tr>
<tr>
<td>(0.005)</td>
<td>(0.006)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Adjusted ( R^2 )</td>
<td>0.231</td>
<td>0.228</td>
</tr>
<tr>
<td>Entity Type FE</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Incorporation State FE</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Location State FE</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Intermediary FE</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>461</td>
<td>461</td>
</tr>
</tbody>
</table>

quality startups. This evidence is consistent with the main implication of my model that the Regulation CF market in current design may not be able to overcome the lemons problem and fund high quality startups.

5. Discussion

Theorem 1 proves that there does not exist a crowdfunding market equilibrium. Here, I suggest an improving market mechanism that would support a crowdfunding market equilibrium.

The main reason for the non-existence of a crowdfunding market equilibrium is that the outsiders’ participation constraint cannot be satisfied. At a crowdfunding market equilibrium, because only the insiders with good signals invest, the outsiders can essentially observe the
aggregate of all the insiders’ private signals by observing the aggregate of all the insiders’ actions. As a result, the outsiders have more information than any single insider, thus giving the outsiders information advantage in inferring the quality of the startup, and thus requiring a stronger participation constraint. As studied in the IPO literature (Rock, 1986), underpricing is an effective method for solving the uninformed investors’ participation constraint problem. However, in the crowdfunding market, underpricing may not be so feasible. As argued before, the crowdfunding market needs to aggregate the insiders’ information to overcome the adverse selection problem. When underpricing, on one hand, could solve outsiders’ participation constraint, on the other hand, it could also destroy the insiders’ information aggregation. Indeed, the insiders can collectively aggregate information only when insiders with different private information make different investment decisions ($G$-signal insiders invest and $B$-signal insiders do not). But because the information difference between $G$-signal and $B$-signal insiders is small if they are strategic, very little room exists for the share price to be set so that insiders with different levels of information invest differently. This limits the practicality of underpricing in the crowdfunding market: lowering the share price by any significant amount will make it worthwhile for $B$-signal investors to deviate, leading insiders’ information aggregation to fail.

To remedy the non-existence of a crowdfunding market equilibrium, one simple mechanism is to set different prices for the insiders and outsiders. More specifically, the entrepreneur can set a state-contingent share price for the outsiders. Consider $n_I$, the number of insiders who invest, as the states. Then a state-contingent price for the outsiders is a price function $p(n_I)$ with $n_0 + 1 \leq n_I \leq n + 1$. The entrepreneur solves a new optimization problem:

$$\max_{p(n_I)} p^*(n_I) \quad \text{s.t.} \quad P(H|n_I, S) - p(n_I) \geq 0,$$

where $P(H|n_I, S) - p(n_I) \geq 0$ is the outsiders’ participation constraint conditioning on $n_I$ insiders invest. The entrepreneur’s optimization problem can be easily solved.

**Proposition 5.** If the entrepreneur were allowed to set a state-contingent price for the outsiders, the entrepreneur would optimally set the share price $p'(n_I) = \frac{1}{1 + \frac{1}{1+2^n}} (n_0 + 1 \leq n_I \leq n + 1)$ for the outsiders. When the entrepreneur sets the optimal price $p^*$ for the insiders, and $p'(n_I)$ for the outsiders, there exists a crowdfunding market equilibrium.

Proposition 5 implies that were the entrepreneur allowed to set a state-contingent price for the outsiders, she can offer a discount on the share price for the outsiders if the realized $n_I$ is low, and charging a premium if the realized $n_I$ is high. Clearly, the state-contingent price in
Proposition 5 satisfies the outsiders’ participation constraint. Under the state-contingent price scheme, the insiders’ information can be conveyed to the outsiders at a market equilibrium, and high quality startups end up with higher probability of being funded.

6. Conclusion

Equity crowdfunding is a new evolving market for startups to raise capital. Like other early-stage financing models, equity crowdfunding also faces high information asymmetries. In this paper, I provide a rigorous analysis for one main type of information asymmetries: adverse selection. I find that no simple market equilibrium exists that could solve the adverse selection problem. Under the current fixed offering price rule, the equity crowdfunding market does not seem able to screen high quality projects. Using Regulation CF filings, I also provide empirical evidence that is consistent with my theoretical findings. The inability to overcome the lemons problem could lead to market failure (Akerlof, 1970). My finding suggests that to promote the development of the equity crowdfunding market, introducing new market mechanisms that can remove the adverse selection barrier may be necessary.

References


A.1. Proof of Theorem 1.

Proof. The proof of Theorem 1 directly follows from Proposition 4. Proposition 4 suggests that when the first two conditions of the crowdfunding market equilibrium are satisfied, the third one cannot be satisfied. □

A.2. Proof of Lemma 1.

Proof. Because $E(Y|F,S) = P(H|F,S) - p(F = G,B)$, it suffices to prove that $P(H|G,S) > P(H|B,S)$. By Bayes’ rule and the common prior $P(H) = P(L) = 1/2$,

$$P(H|G,S) = \frac{P(G,S|H)}{P(G,S|H) + P(G,S|L)}.$$ 

Now fix one insider, and assume that this insider invests. Then the event $S$ is determined by all other insiders’ private information realization and other investors’ investing strategies. Let $S'$ denote the event that the other investors buy all but one shares ($n$ out of total $n + 1$ shares) offered in the crowdfunding campaign. Then we can write $P(F, S|H) = P(F, S'|H)$ and $P(F, S|L) = P(F, S'|L)$ for $F = G, B$. So,

$$P(H|G, S) = \frac{P(G, S'|H)}{P(G, S'|H) + P(G, S'|L)}.$$ 

Then by the independence of insiders’ information,

$$P(H|G, S) = \frac{P(G|H)P(S'|H)}{P(G|H)P(S'|H) + P(G|L)P(S'|L)} = \frac{P(S'|H)}{P(S'|H) + \frac{P(G|L)P(S'|L)}{P(G|H)}}.$$ 

Since $\frac{P(G|L)}{P(G|H)} < \frac{P(B|L)}{P(B|H)}$ for $\alpha > 1/2$, we have

$$P(H|G, S) > \frac{P(S'|H)}{P(S'|H) + \frac{P(B|L)}{P(B|H)}P(S'|L)} = P(H|B, S).$$ 

This completes the proof. □

Proof. I prove that the outsiders play cutoff strategy for each possibility of insiders’ strategy profile. Since Cases I and IV can be considered special cases of Cases II and IV, and Case III is an intermediate case, I just need to prove the proposition for Cases II and IV.

Case II: in Case II, G-signal insiders use mixed strategy (with mixed probability \( r_G \)), and B-signal insiders do not invest. Let \( m \) denote the number of insiders that outsiders observe. Then the outsiders’ posterior is:

\[
P(H|m) = \frac{P(m|H)P(H)}{P(m|H)P(H) + P(m|L)P(L)} = \frac{1}{1 + \frac{P(m|L)}{P(m|H)}}.
\]

To prove that outsiders play cutoff strategies, it suffices to prove that \( P(H|m) \) is monotonically increasing in \( m \). To do so, I am going to prove that \( \frac{P(m|L)}{P(m|H)} \) is decreasing in \( m \). It is also equivalent to prove that \( P(m|L)P(m+1|H) - P(m|H)P(m+1|L) > 0 \). Because

\[
P(m|L) = \sum_{k \geq m} \binom{n}{k} (1 - \alpha)^k \alpha^{n-k} \binom{k}{m} P_H^m (1 - r_G)^{k-m}
\]

and

\[
P(m|H) = \sum_{k \geq m} \binom{n}{k} \alpha^k (1 - \alpha)^{n-k} \binom{k}{m} P_H^m (1 - r_G)^{k-m},
\]

I denote \( p_k = \binom{n}{k} (1 - \alpha)^k \alpha^{n-k}, q_k = \binom{n}{k} \alpha^k (1 - \alpha)^{n-k} \) and \( x_k(m) = \binom{k}{m} P_H^m (1 - r_G)^{k-m} \). Then

\[
\frac{P(m|L)}{P(m|H)} = \frac{P(m|L)P(m+1|H) - P(m|H)P(m+1|L)}{P(m|H)P(m+1|L)}
\]

\[
= \left[ \sum_{k \geq m} p_k x_k(m) \right] \cdot \left[ \sum_{k \geq m+1} q_k x_k(m+1) \right] - \left[ \sum_{k \geq m} q_k x_k(m) \right] \cdot \left[ \sum_{k \geq m+1} p_k x_k(m+1) \right]
\]

\[
= \sum_{i \geq m, j \geq m+1} (p_i q_j - q_i p_j) x_i(m) x_j(m+1)
\]

\[
= \sum_{i \geq m+1, j \geq m+1} (p_i q_j - q_i p_j) x_i(m) x_j(m+1)
\]

\[
= \sum_{i=m, j \geq m+1} (p_i q_j - q_i p_j) x_i(m) x_j(m+1) \quad \text{(1)}
\]

\[
+ \sum_{i=m, j \geq m+1} (p_i q_j - q_i p_j) x_i(m) x_j(m+1) \quad \text{(2)}
\]

Then it suffices to prove that both (1) and (2) are positive. To prove that (1) is positive, fix a pair \( i, j \) with \( i < j \). Observe that

\[
x_i(m) x_j(m+1) = \binom{i}{m} P_H^m (1 - r_G)^{i-m} \binom{j}{m+1} P_H^{m+1} (1 - r_G)^{j-m-1}
\]
Also note that \( p_i q_j - q_i p_j > 0 \) for \( i < j \). Thus,

\[
(p_i q_j - q_i p_j) x_i(m) x_j(m + 1) > (p_i q_j - q_i p_j) x_j(m) x_i(m + 1).
\]

So

\[
(p_i q_j - q_i p_j) x_i(m) x_j(m + 1) + (p_j q_i - q_j p_i) x_j(m) x_i(m + 1) > 0.
\]

Then by the symmetry of pairs of summands in (1), (1) is positive. Since \( p_i q_j - q_i p_j > 0 \) for \( i < j \), (2) is also positive. Thus, \( \mathbb{P}(m|L)\mathbb{P}(m + 1|H) - \mathbb{P}(m|H)\mathbb{P}(m + 1|L) > 0 \). This proves Case II.

**Case IV:** in Case IV, \( G \)-signal insiders invest, and \( B \)-signal insiders use mixed strategy (with mixed probability \( r_B \)). Similarly, I just need to prove that \( \mathbb{P}(m|L)\mathbb{P}(m + 1|H) - \mathbb{P}(m|H)\mathbb{P}(m + 1|L) > 0 \). Because

\[
\mathbb{P}(m|L) = \sum_{k \leq m} \binom{n}{k} (1 - \alpha)^k \alpha^{n-k} \binom{n-k}{m-k} r_B^{m-k}(1 - r_B)^{n-m}
\]

and

\[
\mathbb{P}(m|H) = \sum_{k \leq m} \binom{n}{k} \alpha^k (1 - \alpha)^{n-k} \binom{n-k}{m-k} r_B^{m-k}(1 - r_B)^{n-m},
\]

I denote \( p_k = \binom{n}{k} (1 - \alpha)^k \alpha^{n-k} \), \( q_k = \binom{n}{k} \alpha^k (1 - \alpha)^{n-k} \) and \( x_k(m) = \binom{n-k}{m-k} r_B^{m-k}(1 - r_B)^{n-m} \). Then

\[
\mathbb{P}(m|L)\mathbb{P}(m + 1|H) - \mathbb{P}(m|H)\mathbb{P}(m + 1|L)
\]

\[
= \left[ \sum_{k \leq m} p_k x_k(m) \right] \cdot \left[ \sum_{k \leq m+1} q_k x_k(m + 1) \right] - \left[ \sum_{k \leq m} q_k x_k(m) \right] \cdot \left[ \sum_{k \leq m+1} p_k x_k(m + 1) \right]
\]

\[
= \sum_{i \leq m, j \leq m + 1} p_i q_j x_i(m) x_j(m + 1) - \sum_{i \leq m, j \leq m + 1} q_i p_j x_i(m) x_j(m + 1)
\]

\[
= \sum_{i \leq m, j \leq m + 1} (p_i q_j - q_i p_j) x_i(m) x_j(m + 1)
\]
receives zero expected payoff from investing, i.e. $E_B$ between investing and not investing. This gives the equilibrium condition that a

At Equilibrium II, since the $G$-signal insiders play mixed strategy, they should be indifferent between investing and not investing. This particularly implies that there is a unique symmetric equilibrium strategy for the insiders. Now, to prove Proposition 2, it suffices to find the ranges of $P(H|G, S)$ and $P(H|B, S)$.
which identify the price $p$ such that there exists a corresponding insiders’ equilibrium strategy. The rest of this proof is to find the ranges of $\mathbb{P}(H|G,S)$ and $\mathbb{P}(H|B,S)$.

(1) The proof of Part I trivially follows from Part II.

(2) Notice that $\mathbb{P}(H|G,S)$ is not continuous at $r_G = 0$. Now, to prove Part II, it suffices to prove that 

$$\lim_{r_G \to 0^+} \mathbb{P}(H|G,S) = \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{n_0+1}}.$$ 

By Lemma 2 below, this holds because 

$$\lim_{r_G \to 0^+} \xi_{II}(r_G, n, n_0, \alpha) = \left(\frac{1-\alpha}{\alpha}\right)^{n_0}.$$ 

(3) I just need to prove $\mathbb{P}(H|B,S)|_{r_B=0} < \mathbb{P}(H|G,S)|_{r_G=1}$. Notice that $r_G = 1$ corresponds to the insiders’ equilibrium strategy that $G$-signal insiders invest with mixed probability $r_G = 1$ and $B$-signal insiders do not invest, and $r_B = 0$ corresponds to the insiders’ equilibrium strategy that $G$-signal insiders invest and $B$-signal insiders invest with mixed probability $r_B = 0$. In both cases, only $G$-signal investors invest, and thus the two quantities $\mathbb{P}(n_i \geq n_0|L)$ and $\mathbb{P}(n_i \geq n_0|H)$ are the same. So to prove $\mathbb{P}(H|B,S)|_{r_B=0} < \mathbb{P}(H|G,S)|_{r_G=1}$, it suffices to prove $\frac{\alpha}{1-\alpha} > \frac{1-\alpha}{\alpha}$. This clearly holds for any $\alpha > 1/2$.

(4) Since $\mathbb{P}(H|B,S)$ is continuous in $r_B \in [0,1]$, it is straightforward to identify the range of $\mathbb{P}(H|B,S)$, which then proves Part IV.

(5) The proof of Part E trivially follows from Part IV.

□


Proof. Part (1): I first calculate $\mathbb{P}(H|G,S)$, a $G$-signal insider’s gross payoff from investing when all other insiders play equilibrium strategies of Equilibrium II. By Bayes’ rule and the common prior $\mathbb{P}(H) = \mathbb{P}(L) = 1/2$, 

$$\mathbb{P}(H|G,S) = \frac{\mathbb{P}(G,S|H)}{\mathbb{P}(G,S|H) + \mathbb{P}(G,S|L)}.$$ 

Since the outsiders are playing equilibrium strategy $n_0 + 1$, the event $S$ is equivalent to that at least $n_0 + 1$ insiders invest. For one insider, let $n_i$ denote the number of other insiders who invest. Then for one insider, conditioning on her own investing decision, $n_i = n_I - 1$, and $S$ is
also equivalent to the event that there are at least \( n_0 \) other insiders who invest, i.e. \( n_i \geq n_0 \). So,

\[
\mathbb{P}(H|G, S) = \frac{\mathbb{P}(G, n_i \geq n_0|H)}{\mathbb{P}(G, n_i \geq n_0|H) + \mathbb{P}(G, n_i \geq n_0|L)}.
\]

By the independence of insiders’ private information, I can rewrite the above equation as follows:

\[
\mathbb{P}(H|G, S) = \frac{1}{1 + \frac{\mathbb{P}(G|L)}{\mathbb{P}(G|H)} \cdot \frac{\mathbb{P}(n_i > n_0|L)}{\mathbb{P}(n_i > n_0|H)}}.
\]

At Equilibrium II,

\[
\mathbb{P}(n_i \geq n_0|H) = \sum_{k \geq n_0} \mathbb{P}(n_i \geq n_0|k) \mathbb{P}(k \text{ \text{G signals} } |H)
\]

\[
= \sum_{k \geq n_0} \left( \sum_{n_i \geq n_0} \left( \begin{array}{c} k \\ n_i \end{array} \right) r_G^{n_i}(1 - r_G)^{k-n_i} \right) \left( \begin{array}{c} n \\ k \end{array} \right) \alpha^k (1 - \alpha)^{n-k},
\]

and

\[
\mathbb{P}(n_i \geq n_0|L) = \sum_{k \geq n_0} \left( \sum_{n_i \geq n_0} \left( \begin{array}{c} k \\ n_i \end{array} \right) r_G^{n_i}(1 - r_G)^{k-n_i} \right) \left( \begin{array}{c} n \\ k \end{array} \right) (1 - \alpha)^{k \alpha^{n-k}}.
\]

so \( \mathbb{P}(H|G, S) \) can be expressed a function of parameters \( n, n_0, \alpha, r_G \):

\[
\mathbb{P}(H|G, S) = \frac{1}{1 + \frac{\sum_{k \geq n_0} \left( \sum_{n_i \geq n_0} \left( \begin{array}{c} k \\ n_i \end{array} \right) r_G^{n_i}(1 - r_G)^{k-n_i} \right) \left( \begin{array}{c} n \\ k \end{array} \right) (1 - \alpha)^{k \alpha^{n-k}}}{\sum_{k \geq n_0} \left( \sum_{n_i \geq n_0} \left( \begin{array}{c} k \\ n_i \end{array} \right) r_G^{n_i}(1 - r_G)^{k-n_i} \right) \left( \begin{array}{c} n \\ k \end{array} \right) \alpha^k (1 - \alpha)^{n-k}} \cdot \frac{\mathbb{P}(n_i > n_0|L)}{\mathbb{P}(n_i > n_0|H)} \cdot \frac{\mathbb{P}(G|L)}{\mathbb{P}(G|H)}.
\]

For fixed parameters \( n, n_0, \alpha, \mathbb{P}(H|G, S) \) is a function of \( r_G \), which reflects how the gross payoff of a \( G \)-signal insider varies with his equilibrium strategy. Let

\[
\xi_{II}(r_G, n, n_0, \alpha) = \frac{\mathbb{P}(n_i \geq n_0|L)}{\mathbb{P}(n_i \geq n_0|H)} = \frac{\sum_{k \geq n_0} \left( \sum_{n_i \geq n_0} \left( \begin{array}{c} k \\ n_i \end{array} \right) r_G^{n_i}(1 - r_G)^{k-n_i} \right) \left( \begin{array}{c} n \\ k \end{array} \right) (1 - \alpha)^{k \alpha^{n-k}}}{\sum_{k \geq n_0} \left( \sum_{n_i \geq n_0} \left( \begin{array}{c} k \\ n_i \end{array} \right) r_G^{n_i}(1 - r_G)^{k-n_i} \right) \left( \begin{array}{c} n \\ k \end{array} \right) \alpha^k (1 - \alpha)^{n-k}}.
\]

To prove the first part of Lemma 2, it suffices to prove that for any fixed \( n, n_0, \alpha, \xi_{II}(r_G, n, n_0, \alpha) \) is strictly increasing in \( r_G \). I will prove that \( \frac{\partial \xi_{II}(r_G, n, n_0, \alpha)}{\partial r_G} > 0 \).

Rewrite \( \xi_{II}(r_G, n, n_0, \alpha) \) as

\[
\xi_{II}(r_G, n, n_0, \alpha) = \frac{\sum_{k \geq n_0} \left[ n_0 \left( \begin{array}{c} k \\ n_0 \end{array} \right) \int_0^{r_G} t^{n_0-1}(1-t)^{k-n_0} dt \right) \left( \begin{array}{c} n \\ k \end{array} \right) (1 - \alpha)^{k \alpha^{n-k}}}{\sum_{k \geq n_0} \left[ n_0 \left( \begin{array}{c} k \\ n_0 \end{array} \right) \int_0^{r_G} t^{n_0-1}(1-t)^{k-n_0} dt \right] \left( \begin{array}{c} n \\ k \end{array} \right) \alpha^k (1 - \alpha)^{n-k}}.
\]
Let \( a_k = n_0(k) \int_0^{r_G} t^{n_0-1}(1-t)^{k-n_0} dt \). Then \( a_k \)'s derivative with respect to \( r_G \) is \( a_k' = n_0(k) r_G^{n_0-1}(1-r_G)^{k-n_0} \). Take partial derivative of \( \xi_{II}(r_G, n, n_0, \alpha) \) with respect to \( r_G \):

\[
\frac{\partial \xi_{II}(r_G, n, n_0, \alpha)}{\partial r_G} = \frac{\sum_{k \geq n_0} \binom{n}{k} a_k' \alpha^k (1-\alpha)^{n-k} \left[ \sum_{k \geq n_0} \binom{n}{k} a_k \alpha^k (1-\alpha)^{n-k} \right] - \sum_{k \geq n_0} \binom{n}{k} a_k' \alpha^k (1-\alpha)^{n-k} \left[ \sum_{k \geq n_0} \binom{n}{k} a_k (1-\alpha)^k \alpha^{n-k} \right]}{\left[ \sum_{k \geq n_0} \binom{n}{k} a_k \alpha^k (1-\alpha)^{n-k} \right]^2}.
\]

To prove (9) is positive, I just need to prove that the nominator is positive. Fix a pair \( i, j \) with \( i < j \). Observe that

\[
a_i a_j' = n_0 \left( \begin{array}{c} i \\ n_0 \end{array} \right) \int_0^{r_G} t^{n_0-1}(1-t)^{i-n_0} dt \cdot n_0 \left( \begin{array}{c} j \\ n_0 \end{array} \right) r_G^{n_0-1}(1-r_G)^{j-n_0} = n_0 \left( \begin{array}{c} i \\ n_0 \end{array} \right) \int_0^{r_G} t^{n_0-1}(1-t)^{i-n_0}(1-r_G)^{j-i} dt \cdot n_0 \left( \begin{array}{c} j \\ n_0 \end{array} \right) r_G^{n_0-1}(1-r_G)^{j-n_0} < n_0 \left( \begin{array}{c} i \\ n_0 \end{array} \right) \int_0^{r_G} t^{n_0-1}(1-t)^{i-n_0} dt \cdot n_0 \left( \begin{array}{c} j \\ n_0 \end{array} \right) r_G^{n_0-1}(1-r_G)^{j-n_0} = a_i a_j'.
\]

Then for any fixed pair \( i, j \) with \( i < j \), the inequality holds

\[
a_i a_j' \alpha^n (1-\alpha)^n \left[ \alpha^{i-j}(1-\alpha)^{j-i} - \alpha^{j-i}(1-\alpha)^{i-j} \right] + a_j a_i' \alpha^n (1-\alpha)^n \left[ \alpha^{j-i}(1-\alpha)^{i-j} - \alpha^{i-j}(1-\alpha)^{j-i} \right] > a_i a_j' \alpha^n (1-\alpha)^n \left[ \alpha^{i-j}(1-\alpha)^{j-i} - \alpha^{j-i}(1-\alpha)^{i-j} \right] + a_j a_i' \alpha^n (1-\alpha)^n \left[ \alpha^{j-i}(1-\alpha)^{i-j} - \alpha^{i-j}(1-\alpha)^{j-i} \right] = 0.
\]

So the nominator of (9) is positive. This proves \( \frac{\partial \xi_{II}(r_G, n, n_0, \alpha)}{\partial r_G} > 0 \), and thus part (1) of Lemma 2. Since \( \xi_{II}(r_G, n, n_0, \alpha) \) is monotone, we can easily calculate its limits with respect to \( r_G \):

\[
\lim_{r_G \to 0^+} \xi_{II}(r_G, n, n_0, \alpha) = \lim_{r_G \to 0^+} \frac{\sum_{k \geq n_0} n_0(k) r_G^{n_0-1}(1-r_G)^{k-n_0} \left( \begin{array}{c} n \\ k \end{array} \right) (1-\alpha)^k \alpha^{n-k}}{\sum_{k \geq n_0} n_0(k) r_G^{n_0-1}(1-r_G)^{k-n_0} \left( \begin{array}{c} n \\ k \end{array} \right) (1-\alpha)^k \alpha^{n-k}} = \frac{\sum_{k \geq n_0} n_0(k) \left( \begin{array}{c} n \\ k \end{array} \right) (1-\alpha)^k \alpha^{n-k}}{\sum_{k \geq n_0} n_0(k) \left( \begin{array}{c} n \\ k \end{array} \right) (1-\alpha)^k \alpha^{n-k}}.
\]
For fixed parameters \( b \) and \( \partial \xi \), let \( \xi_{IV}(r_G; n, n_0, \alpha) = \frac{\sum_{k \geq n_0} \binom{n}{k} (1 - \alpha)^k \alpha^{n-k}}{\sum_{k \geq n_0} \binom{n}{k} \alpha^k (1 - \alpha)^{n-k}} \).

\[
\lim_{r_G \to 1^-} \xi_{IV}(r_G; n, n_0, \alpha) = \frac{\sum_{k \geq n_0} \binom{n}{k} (1 - \alpha)^k \alpha^{n-k}}{\sum_{k \geq n_0} \binom{n}{k} \alpha^k (1 - \alpha)^{n-k}}.
\]

**Part (2):** Similarly as before, I can express \( \mathbb{P}(H|B, S) \) as a function of parameters \( n, n_0, \alpha, r_B \):

\[
\mathbb{P}(H|B, S) = \frac{1}{1 + \frac{\mathbb{P}(n_i \geq n_0|L)}{\mathbb{P}(n_i \geq n_0|H)}},
\]

For fixed parameters \( n, n_0, \alpha, \mathbb{P}(H|B, S) \) as a function of \( r_B \), reflects how the gross payoff of a \( B \)-signal insider varies with his equilibrium strategy. Similarly, let \( \xi_{IV}(r_B; n, n_0, \alpha) := \frac{\mathbb{P}(n_i \geq n_0|L)}{\mathbb{P}(n_i \geq n_0|H)} \) be a function of \( n, n_0, \alpha, r_B \). Then to prove Part (2) of Lemma 2, it suffices to prove that for any fixed \( n, n_0, \alpha, \xi_{IV}(r_B, n, n_0, \alpha) \) is strictly increasing in \( r_B \). I will prove that \( \frac{\partial \xi_{IV}(r_B, n, n_0, \alpha)}{\partial r_B} > 0 \).

Let \( b_k = \mathbb{P}(n_i \geq n_0|k \text{ } B \text{ signals}) \), and \( b_k' \) be its derivative with respect to \( r_B \). Notice that \( b_k = 1 \) for \( k \leq n - n_0 \) at Equilibrium IV. So \( b_k' = 0 \) for \( k \leq n - n_0 \). Taking partial derivative of \( \xi_{IV}(r_B, n, n_0, \alpha) \) with respect to \( r_B \), I have

\[
\frac{\partial \xi_{IV}(r_B, n, n_0, \alpha)}{\partial r_B} = \sum_{i,j \geq 0, i \neq j} \binom{n}{i,j} b_i b_j' \left[ (1 - \alpha)^i \alpha^{n-i} \alpha^j (1 - \alpha)^{n-j} - \alpha^i (1 - \alpha)^{n-i} (1 - \alpha)^j \alpha^{n-j} \right]
\]

\[
= \sum_{i,j \geq 0, i \neq j} \binom{n}{i,j} b_i b_j' \left[ (1 - \alpha)^i \alpha^{n-i} \alpha^j (1 - \alpha)^{n-j} - (1 - \alpha)^{j-i} \alpha^{i-j} \right]
\]

\[
= \frac{\sum_{k \geq 0} \binom{n}{k} b_k(1 - \alpha)^k \alpha^{n-k}}{\sum_{k \geq 0} \binom{n}{k} \alpha^k (1 - \alpha)^{n-k}}.
\]

\[
(11)
\]

It suffices to prove that the nominator of (11) is positive.

Let \( n_1 = n - n_0 \). For \( k \geq n - n_0 + 1 \), write \( b_k = (n_0 - (n - k)) \binom{k}{n_0 - (n - k)} \int_0^{r_B} t^{n_0 - (n - k) - 1} (1 - t)^{k - n_0 + (n - k)} dt \) = \( (n_1 - 1) \binom{k}{n_1} \int_0^{r_B} (1 - t)^{n_1 - 1} t^{k - n_1 - 1} (1 - t)^{n_1} dt \). Then \( b_k' = (k - n_1) \binom{k}{n_1} \int_0^{r_B} (1 - t)^{n_1} (1 - r_B)^{n_1} \).

Fix a pair of \( i, j \) with \( n - n_0 + 1 \leq i < j \). Then,

\[
b_i b_j' = (i - n_1) \binom{i}{i - n_1} \int_0^{r_B} t^{j - n_1 - 1} (1 - t)^{n_1} dt \cdot (j - n_1) \binom{j}{j - n_1} \int_0^{r_B} (1 - t)^{n_1} (1 - r_B)^{n_1} \]

\[
= (i - n_1) \binom{i}{i - n_1} \int_0^{r_B} t^{j - n_1 - 1} r_B^{j - n_1 - 1} (1 - t)^{n_1} dt \cdot (j - n_1) \binom{j}{j - n_1} \int_0^{r_B} (1 - t)^{n_1} (1 - r_B)^{n_1} \]
> (i - n_1) \int_0^{r_B} t^{i-n_1-1} (1-t)^{n_1} dt \cdot (j - n_1) \int_0^{r_B} t^{j-n_1-1} (1-t)^{n_1} dt 
= b_j b'_i.

Then for any fixed pair \(i, j\) with \(n - n_0 + 1 \leq i < j\), the sum of the two terms indexed by \((i, j)\) and \((j, i)\) in the nominator of (11) is positive:

\[
\text{Summand}_{(i,j)} + \text{Summand}_{(j,i)} = \binom{n}{i} \binom{n}{j} b_j b'_i \alpha^n (1-\alpha)^n [(1-\alpha)^{i-j} \alpha^{i-j} - (1-\alpha)^{j-i} \alpha^{i-j}] + \binom{n}{j} \binom{n}{i} b_j b'_i \alpha^n (1-\alpha)^n [(1-\alpha)^{j-i} \alpha^{i-j} - (1-\alpha)^{i-j} \alpha^{j-i}]
\]

For \(i < n - n_0 + 1\) and \(j \geq n - n_0 + 1\), it also holds that

\[
\binom{n}{i} \binom{n}{j} b_j b'_i \alpha^n (1-\alpha)^n [(1-\alpha)^{i-j} \alpha^{i-j} - (1-\alpha)^{j-i} \alpha^{i-j}] > 0.
\]

So the nominator of (11) is positive, and \(\frac{\partial \xi_{IV}(r_B, n, n_0, \alpha)}{\partial r_B} > 0\). This proves part (2) of Lemma 2. \(\square\)


**Proof.** Write

\[
IA = \frac{\mathbb{P}(S|H)}{\mathbb{P}(S|L)} = \frac{\mathbb{P}(n_i \geq n_0|H)}{\mathbb{P}(n_i \geq n_0|L)}
\]

Then at Equilibrium II, \(IA = \frac{1}{\xi_{II}(r_G, n, n_0, \alpha)}\). As proved in the proof of Lemma 2, \(\xi_{II}(r_G, n, n_0, \alpha)\) is strictly increasing in \(r_G\), so \(IA\) is strictly decreasing in \(r_G\) at insiders’ Equilibrium II. Similarly, at Equilibrium IV, \(IA = \frac{1}{\xi_{IV}(r_B, n, n_0, \alpha)}\), and \(IA\) is strictly decreasing in \(r_B\). \(\square\)


**Proof.** Let \(R_\gamma(r_G)\) denote the entrepreneur’s expected utility. So for \(\gamma \in (0, 1]\),

\[
R_\gamma(r_G) := \frac{\mathbb{P}(n_i \geq n_0|H) + \mathbb{P}(n_i \geq n_0|L)}{2} \cdot p^\gamma(r_G)
\]
To prove Propositions 3, it suffices to prove that \( R_\gamma(r_G) \) is increasing in \( r_G \). I will prove that \( \frac{\partial R_\gamma(r_G)}{\partial r_G} > 0 \). Calculate the partial derivative of \( R_\gamma(r_G) \) with respect to \( r_G \):

\[
\frac{\partial R_\gamma(r_G)}{\partial r_G} = \frac{\partial}{\partial r_G} \left[ \frac{\mathbb{P}(n_i \geq n_0|H) + \mathbb{P}(n_i \geq n_0|L)}{2} \cdot p_\gamma(r_G) \right] + \frac{\partial}{\partial r_G} \left[ \frac{\mathbb{P}(n_i \geq n_0|H) + \mathbb{P}(n_i \geq n_0|L)}{2} \cdot \gamma p_\gamma^{-1}(r_G) \cdot p'(r_G) \right] = \frac{\partial}{\partial r_G} \left[ \frac{\mathbb{P}(n_i \geq n_0|H) + \mathbb{P}(n_i \geq n_0|L)}{2} \cdot p_\gamma(r_G) \right] + \frac{\partial}{\partial r_G} \left[ \frac{\mathbb{P}(n_i \geq n_0|H) + \mathbb{P}(n_i \geq n_0|L)}{2} \cdot \gamma p_\gamma^{-1}(r_G) \cdot \left( -p_\gamma^{-1}(r_G) \frac{1-\alpha}{\alpha} \cdot \xi_{II}(r_G, n, n_0, \alpha) \right) \right].
\]

Let \( t = \frac{\mathbb{P}(n_i \geq n_0|H)}{\mathbb{P}(n_i \geq n_0|H)} \cdot \mathbb{P}(n_i \geq n_0|H) \). Observe that \( t > 0 \). Then

\[
\frac{\partial R_\gamma(r_G)}{\partial r_G} = \frac{p_\gamma^{-1}(r_G)\mathbb{P}(n_i \geq n_0|H)}{2} \cdot \left[ \frac{1 + t}{p(r_G)} - (1 + \xi_{II}(r_G, n, n_0, \alpha)) \right] \cdot \gamma \cdot \left( -p_\gamma^{-1}(r_G) \frac{1-\alpha}{\alpha} \cdot \xi_{II}(r_G, n, n_0, \alpha) \right) \right].
\]

By the equation \( \frac{\mathbb{P}(n_i \geq n_0|H)\xi_{II}(r_G, n, n_0, \alpha)}{\mathbb{P}(n_i \geq n_0|H)} = t - \xi_{II}(r_G, n, n_0, \alpha) \),

\[
\frac{\partial R_\gamma(r_G)}{\partial r_G} = \frac{p_\gamma^{-1}(r_G)\mathbb{P}(n_i \geq n_0|H)}{2} \cdot \left[ \frac{1 + t}{p(r_G)} - (1 + \xi_{II}(r_G, n, n_0, \alpha)) \right] \cdot \gamma \cdot \left( -p_\gamma^{-1}(r_G) \frac{1-\alpha}{\alpha} \cdot (t - \xi_{II}(r_G, n, n_0, \alpha)) \right) \right].
\]

Then by the insiders’ equilibrium condition at Equilibrium II,

\[
p(r_G) = \frac{1}{1 + \frac{\mathbb{P}(G|L)}{\mathbb{P}(G|H)} \cdot \frac{\mathbb{P}(n_i \geq n_0|L)}{\mathbb{P}(n_i \geq n_0|H)}} = \frac{1}{1 + \frac{1-\alpha}{\alpha} \cdot \xi_{II}(r_G, n, n_0, \alpha)}.
\]

Replace \( p(r_G) \) in the expression of \( \frac{\partial R_\gamma(r_G)}{\partial r_G} \):

\[
\frac{\partial R_\gamma(r_G)}{\partial r_G} = \frac{p_\gamma^{-1}(r_G)\mathbb{P}(n_i \geq n_0|H)}{2} \cdot \left[ (1 + t) \left( 1 + \frac{1-\alpha}{\alpha} \xi_{II} \right) - (1 + \xi_{II}) \cdot \gamma \cdot \left( \frac{1-\alpha}{\alpha} \cdot (t - \xi_{II}) \right) \right] \]

\[
= \frac{p_\gamma^{-1}(r_G)\mathbb{P}(n_i \geq n_0|H)}{2} \cdot \left[ 1 + (1 + \gamma) \frac{1-\alpha}{\alpha} \xi_{II} + \gamma \frac{1-\alpha}{\alpha} \xi_{II}^{-2} + \left( 1 - \gamma \frac{1-\alpha}{\alpha} \right) t + (1 - \gamma) \frac{1-\alpha}{\alpha} \xi_{II} t \right] > 0.
\]

So this completes the proof. □

Proof. After observing the number of insiders who invest (denoted by \( n_I \)), the outsiders update their beliefs about the quality of the startup:

\[
\mathbb{P}(H|n_I) = \frac{\mathbb{P}(n_I|H)\mathbb{P}(H)}{\mathbb{P}(n_I|H)\mathbb{P}(H) + \mathbb{P}(n_I|L)\mathbb{P}(L)} = \frac{\mathbb{P}(n_I|H)}{\mathbb{P}(n_I|H) + \mathbb{P}(n_I|L)} \]

Then if investing, the outsiders’ expected gross payoff is

\[
\mathbb{P}(H|n_I) = \frac{1}{1 + \left(\frac{\alpha}{1-\alpha}\right)^{n+1-2n_I}}.
\]

First notice that \( \mathbb{P}(H|n_I) \) is monotone increasing in \( n_I \). Moreover, since \( \alpha > 1/2 \), the following holds:

\[
\mathbb{P}(H|n_I) \approx \begin{cases} 
0 & \text{if } n_I < \frac{n+1}{2}; \\
1 & \text{if } n_I > \frac{n+1}{2}.
\end{cases}
\]

Then clearly, when \( n_0 \leq \frac{n+1}{2} \), \( \mathbb{P}(H|n_0 + 1) - p < 0 \). So it suffices to prove that when \( n_0 \leq \frac{n+1}{2} \), \( \mathbb{P}(H|n_0 + 1) < p \). At Equilibrium II, when the entrepreneur chooses the optimal price, at the threshold \( n_I = n_0 + 1 \) the outsiders will have expected payoff

\[
\mathbb{P}(H|n_0 + 1) - p = \frac{1}{1 + \left(\frac{\alpha}{1-\alpha}\right)^{n+1-2(n_0+1)}} - \frac{1}{1 + \frac{1-\alpha}{\alpha} \cdot e(n, n_0, \alpha)}.
\]  

(12)

To prove \( \mathbb{P}(H|n_0 + 1) < p \), it suffices to prove \( e(n, n_0, \alpha) = \frac{\sum_{k\geq n_0} \binom{n}{k}(1-\alpha)^{k}\alpha^{n-k}}{\sum_{k\geq n_0} \binom{n}{k}(1-\alpha)^{k}\alpha^{n-k}} < \left(\frac{1-\alpha}{\alpha}\right)^{2n_0-n} \). Indeed, this inequality holds because \( \binom{n}{k}(1-\alpha)^{k}\alpha^{n-k} = (1-\alpha)^{2n_0-n} \) for \( k = n_0 \) and \( \binom{n}{k}(1-\alpha)^{k}\alpha^{n-k} = \left(\frac{1-\alpha}{\alpha}\right)^{2k-n} \) is strictly decreasing in \( k \) for \( k \geq n_0 \).


Proof. Fix any \( n_0 \) such that \( n_0 > \frac{n+1}{2} \). Then there exists a crowdfunding market equilibrium in which (1) the entrepreneur sets the share price \( p^* = \frac{1}{1 + \frac{1-\alpha}{\alpha} \cdot e(n, n_0, \alpha)} \) for the insiders, and \( p'(n_I) = \frac{1}{1 + \left(\frac{\alpha}{1-\alpha}\right)^{n+1-2n_I}} \) for the outsiders, (2) the insiders with good signal invest and with bad signal do not invest, and (3) the outsider play equilibrium strategy \( n_0 + 1 \). The completes the proof. 

\[\square\]